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A note on the stability of developing laminar pipe flow subjected to axisymmetric and non-axisymmetric disturbances

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This paper presents a summary of the results of an extensive experimental investigation of the problem considered by Tatsumi (1952*a*, *b*) and more recently by Huang & Chen (1974*a*, *b*). The results, like the analyses, show that the linear instability is confined to the non-similar inlet region of the pipe.

The linear stability analyses of fully developed Poiseuille flow have shown that (i) the flow is stable to all axisymmetric and non-axisymmetric small disturbances (see, for example, Corcos & Sellars 1959; Garg & Rouleau 1972; among others), (ii) the flow is less stable to non-axisymmetric small disturbances with azimuthal wavenumber $n = 1$ than to axisymmetric small disturbances ($n = 0$), and that (iii) the flow is *unstable* to swirl induced by rotating the pipe (Mackrodt 1971). This led Mackrodt to suggest that the instability observed in classical Poiseuille flow may be due to swirl introduced from the reservoir. He did not, however, examine the stability of developing pipe flow to imposed swirl.

Tatsumi (1952*a*, *b*), anticipating instability of boundary-layer type, carried out a numerical analysis of the stability characteristics of the hydrodynamically developing region of the flow for the least stable axisymmetric disturbances using 'almost similar' velocity profiles and an asymptotic method of solution. He obtained a minimum critical Reynolds number $R_c = \overline{U}D/\nu$ of

$$19400 \quad \text{at} \quad x/DR_c = 0{\cdot}375 \times 10^{-3},$$

where D is the pipe diameter and $\overline{U}$ the mean velocity.

Huang & Chen (1974*a*, *b*) re-examined the stability of developing pipe flow to axisymmetric as well as non-axisymmetric small disturbances. The main-flow velocity field was represented by the solution given by Sparrow, Lin & Lundgren (1964) and the eigenvalue problem was solved by a direct numerical integration scheme along with an iteration procedure. The results of Huang & Chen, which are thought to be more accurate and reliable than those of Tatsumi, yielded $R_c = 39800$ at $x/DR_c = 0{\cdot}8125 \times 10^{-3}$ for the axisymmetric disturbances and $R_c = 39560$ at $x/DR_c = 1{\cdot}225 \times 10^{-3}$ for the non-axisymmetric case. Evidently, the response of the developing flow to each type of disturbance is in accord with the character of the flow. Very near the inlet, the flow is very close to being a plane flow of boundary-layer type and thus Squire's (1933) theorem applies. Further downstream from the inlet, axisymmetry dominates and Squire's theorem does not hold (Sexl & Spielberg 1958), i.e. plane waves are no longer more critical for the flow than disturbances of a three-dimensional nature.

No measurements appear to be available for comparison with these calculations. Observations on turbulent slugs in pipe flow by Sarpkaya (1966*a*), Lindgren (1957) and Wygnanski & Champagne (1973) have shown that turbulent bursts and spots always originate in the entrance region as a consequence of amplified free-stream disturbances. This, in fact, led Wygnanski & Champagne to suggest that "stability calculations in the developing region of the pipe are more relevant to the natural process of transition in a pipe (Tatsumi 1952*a*, *b*) than the numerous analyses which are solely concerned with the stability of Poiseuille flow".

Impetus for the present experimental work (initiated in 1965) came partly from Tatsumi's analysis and partly from the observations of turbulent spots in the non-similar inlet regions of pipes. Preliminary results for the axisymmetric disturbances were first reported in Sarpkaya (1966*b*). The following is a brief description of the entire work and a comparison of the results with those obtained by Tatsumi (1952*a*, *b*) and Huang & Chen (1974*a*, *b*).

Experiments were conducted in a flow system (see figure 1, plate 1) which consisted of a silica gel dryer, filters, rotameters, a large settling chamber, porous foam flow straighteners, disturbance generators, a hot-wire anemometer system, a 75 ft long lucite pipe 1·5 in. diameter and a suction pump (separately mounted on flexible neoprene mountings and connected to the pipe by a flexible rubber tube). The entire system was mounted on large concrete blocks placed 10 ft apart and dynamically isolated from the laboratory floor by air springs.

Air flowed from the large settling chamber into the pipe through a specially designed and carefully machined bell-mouth transition. The first 25 ft section of the pipe (entrance section) was interchanged numerous times with pipes of identical size and length for the purpose of installing different disturbance generators or for changing the location of a given disturbance generator. In each entrance section, holes $\frac{1}{8}$ in. in diameter were drilled at suitable intervals and then fitted with lucite screws which, when properly aligned, were flush with the inner surface of the pipe. A hole (0·03 in. in diameter) was provided in the middle of each screw for the insertion of the hot-wire holder before the particular screw was set in place. Considerable attention was given to avoiding surface discontinuities and gaps at the joints. In fact, the test sections were specially manufactured, carefully honed and aligned with extreme precision.

Three types of disturbance generators were used. The first consisted of an electromagnetic device similar to that used by Leite (1959) which could introduce velocity fluctuations into the flow within the pipe at any desired location. The motive element consisted of a vibration meter and magnetic oscillator. When this was excited by an alternating current, a sinusoidal oscillation, parallel to the axis of the pipe, was imparted to the brass sleeve (1·5 in. in diameter, 0·005 in. thick and 1·25 in. long) placed in the pipe. When in position and at rest the sleeve formed a smooth section of the inner wall and did not introduce undesirable disturbances; this was satisfactorily verified by radial and azimuthal surveys of the turbulence at various stations downstream from the sleeve and by comparing the free-stream turbulence levels obtained with and without the sleeve. The amplitude of the oscillations at a given frequency was varied by varying the driving current. The range of frequencies within which the generator operated satisfactorily was from approximately 5 to 200 Hz. No satisfactory procedure was found for ascertaining quickly, prior to each measurement, whether the disturbances generated by the sleeve were completely axisymmetric. Thus, a method had to be devised to see whether the representative disturbances possessed the same general character for different azimuthal angles at each station. For this purpose, the 25 ft section of the test pipe downstream from the disturbance generator was rotated in steps of 5° and u'/U (u' being the r.m.s. downstream velocity fluctuation and U the main-flow velocity) was measured at suitable radial distances for each azimuthal setting at various test sections and at selected Reynolds numbers and disturbance amplitudes and frequencies. Plots of u'/U versus r/a ($a = \frac{1}{2}D$) for all azimuthal positions showed that the disturbances retained axial symmetry and possessed the same general character for different azimuthal angles at each test section. This fact as well as the occurrence of the maximum velocity at the pipe axis indicated the absence of swirl within the pipe. The disturbances generated did not become nonlinear within the range of Reynolds numbers and disturbance amplitudes and frequencies encountered. With this information the making of radial surveys of the disturbances was simplified because the results obtained at various stations could be compared, particularly when the probe was placed at the same azimuthal angle at each station.

Small deviations of the disturbances from axial symmetry could have been caused by the deviation of the fully developed flow from axial symmetry, small leaks at the disturbance generator, minute departures of the shape and the instantaneous position of the disturbance element from axial symmetry, etc. Evidently, such deviations were not troublesome in the present study, primarily because of the meticulous choice of the laboratory test conditions.

The second disturbance generator consisted of a similar sleeve which was oscillated harmonically in the azimuthal direction. Finally, the third disturbance generator consisted of two sets of tripping wires inserted and withdrawn from the pipe by two sets of electromagnets (for details see Sarpkaya 1966*a*).

Measurements of the velocity profiles and free-stream turbulence levels were made at several stations along the pipe, with or without the sleeve, for various Reynolds numbers up to 25000, the maximum Reynolds number below which no transition occurred within the pipe. The turbulence level in the core flow was about 0·7 %. The measurements were made by feeding the signal to an r.m.s. meter and to an amplifier with low and high frequency cut-offs at 0·1 and 1000 Hz respectively. The corresponding critical frequency ranged from 15 to 45 Hz.

The data for the undisturbed velocity profiles were closely represented by

$$U_{\max}/\overline{U} = 2 - \exp\left[-66{\cdot}4x/DR\right]$$

for $x/DR > 0{\cdot}01$. This relationship differs very little from that provided by Langhaar (1940; see Fan & Hwang 1966 for a fairly complete bibliography of flow in the hydrodynamic entrance region). The virtual origin of x depended on the Reynolds number for the bell-mouth used and varied from $x/D = -1$ to $x/D = -2$, x being measured from the start of the uniform test pipe.

Wygnanski & Champagne (1973) have shown that the development of the velocity profile in the inlet region depends not only on x/DR, but also on the nature of the input disturbances. In the present study $(u'/U_{\max})_{℄}$ was about half of the smallest core turbulence level encountered by Wygnanski & Champagne. No attempt was made to increase the input disturbance level and to study its effect on the evolution of the velocity field. Efforts were rather concentrated on obtaining and maintaining as low a disturbance level as possible. The resulting main-flow field agreed with that given by Sparrow *et al.* (1964) to within approximately 5 %. The error in repeatibility was of the order of 3 %.

Measurement of the disturbed laminar entrance flow involved placing a given disturbance generator at a desired location, surveying the field at the remaining downstream stations (the probe being placed at the same azimuthal angle at each section) for various frequencies of oscillation and Reynolds numbers (increased in steps of 500 or 1000, depending on the situation) and repeating the procedure for other disturbance-generator locations and for the other two disturbance generators. This arduous task was then repeated in the fully developed region of the pipe in a manner similar to that of Leite (1959) to verify that those disturbances which were amplified in the entrance region were always damped when introduced into the fully developed region.

The growth or decay of the u component of the disturbances was determined by plotting $\log(u'/u'_0)$ as a function of $(x-x_0)/D$ for various values of r/a, where u'_0 is the r.m.s. amplitude at x_0 (for details see Leite 1959). The rates of growth or decay were nearly linear, particularly for most slowly decaying or growing disturbances. A plot of the growth or decay rates as a function of the Reynolds number for a given location and for the most critical frequency yielded R_c to within ± 500 for the section under consideration. Experiments in the fully developed region of the flow did not show a single case of growth for the disturbances, Reynolds numbers and frequencies used in this investigation.

The results are presented in figure 2 together with those obtained analytically by Tatsumi (1952*a*, *b*) and Huang & Chen (1974*a*, *b*). Although there are considerable differences between the critical Reynolds numbers predicted by Tatsumi and Huang & Chen and those obtained in the present work, all three studies predict that the first instability of the flow occurs in the hydrodynamic entrance region of the pipe. Furthermore, the first instability of the flow seems to occur at about the same axial location. For the axisymmetric disturbances, experiments yield $x/D = 30$ and the analysis (Huang & Chen 1974*a*, *b*) predicts $x/D = 32$. Similar distances for the non-axisymmetric disturbances were found to be 40 and 48 respectively. The differences between the R_c or x/DR_c values may be due to the combined effect of the free-stream turbulence level, the rate of growth of the disturbances, the small differences between the gradients of the measured velocity field and the one used in the analysis, the superposition of some non-

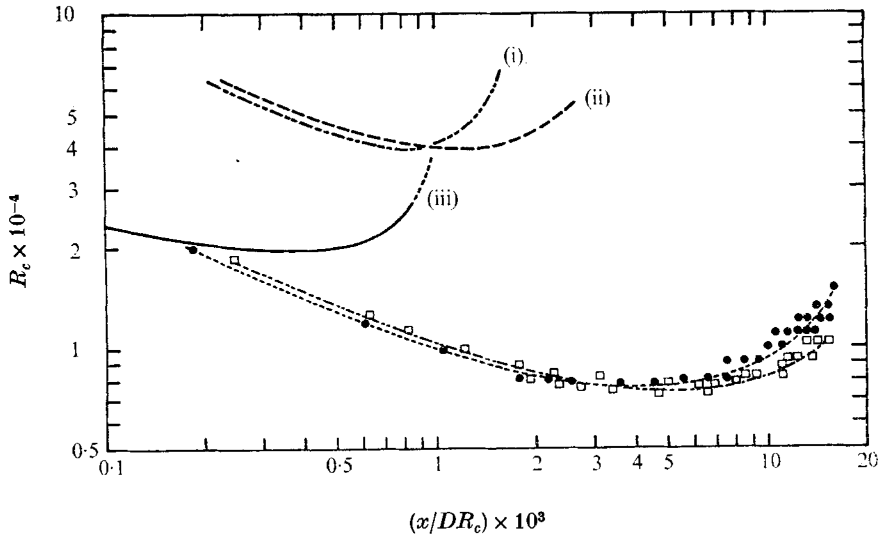

FIGURE 2. Axial variation of the critical Reynolds number. Curves (i) and (ii) show the results of Huang & Chen's (1974*a*, *b*) analysis for axisymmetric and non-axisymmetric disturbances respectively. Curve (iii) shows the result of Tatsumi's (1952*a*, *b*) analysis for axisymmetric disturbances. Experimental data obtained in the present study: ●, axisymmetric disturbances; □, non-axisymmetric disturbances.

axisymmetric disturbances on axisymmetric disturbances, and the manner of introduction of the disturbances. Evidently, the complex localized disturbance introduced experimentally is much more general than the disturbances assumed (along the entire wall) in the mathematical formulation of the problem. The linear theory does not provide an unambiguous choice between the models of spatial and temporal growth. Once, however, nonlinear effects become important, the linear theory is in any case not appropriate, and a flow description cannot be based on that theory. Additional discussion of this important problem may be found in Watson (1962) and in Gaster (1962, 1965). It is, however, important to note that, whether the growth or decay occurs in time or in space, the neutral line is always the same since the two alternatives degenerate to the same analytical problem. Thus, the difference in the values of R_c from analysis and experiments may not be due to the difference in the modes of the disturbances.

An additional comparison between the analytical and experimental results for non-axisymmetric disturbances is presented in table 1, in which α is the axial wavenumber based on the radius of the pipe and c_r is the real part of the wave velocity normalized by the mean velocity. For comparison, the measured physical variables were transformed into the form used in the analysis. Evidently, the experimentally determined values considerably exceed those calculated by Huang & Chen (1974*a*,*b*). Comparison of the factor $(\alpha c_r)_c$ for axisymmetric disturbances also showed an equally unsatisfactory quantitative agreement. Only the trend and the order of magnitude of $(\alpha c_r)_c$ are comparable. It appears that this factor, like the critical Reynolds number, is stretched over larger x/DR_c values. The reasons for the quantitative differences between the

	$(\alpha c_r)_c$	
$(x/DR_c) \times 10^3$	Huang & Chen (1974*a*)	Experimental
0·225	1·50	1·80
0·350	1·21	1·65
0·650	0·91	1·50
0·800	0·82	1·42
0·975	0·75	1·35
1·350	0·63	1·27
1·550	0·58	1·05
2·60	0·39	0·91
3·60	—	0·82
4·60	—	0·75
5·60	—	0·70
6·50	—	0·62
7·50	—	0·57
8·50	—	0·51
9·50	—	0·48
10·0	—	0·43
12·5	—	0·40
15·0	—	0·36

TABLE 1. Analytical and experimental values of the factor $(\alpha c_r)_c$

analytical and experimental results are not yet clear. Similar experiments with lower initial disturbance levels may help to clarify some of these questions which are left unanswered by the present study. Repeating our experiments with the initial disturbance level used does not lead to a better suggestion.

It may be concluded that the data presented herein add a measure of confirmation to the stability trend predicted by Huang & Chen (1974*a*, *b*) for both axisymmetric and non-axisymmetric disturbances and that linear instability is confined to the non-similar inlet region of the pipe. Additional experiments on the evolution of the amplifying disturbances should shed further light on the establishment of turbulent flow in pipes.

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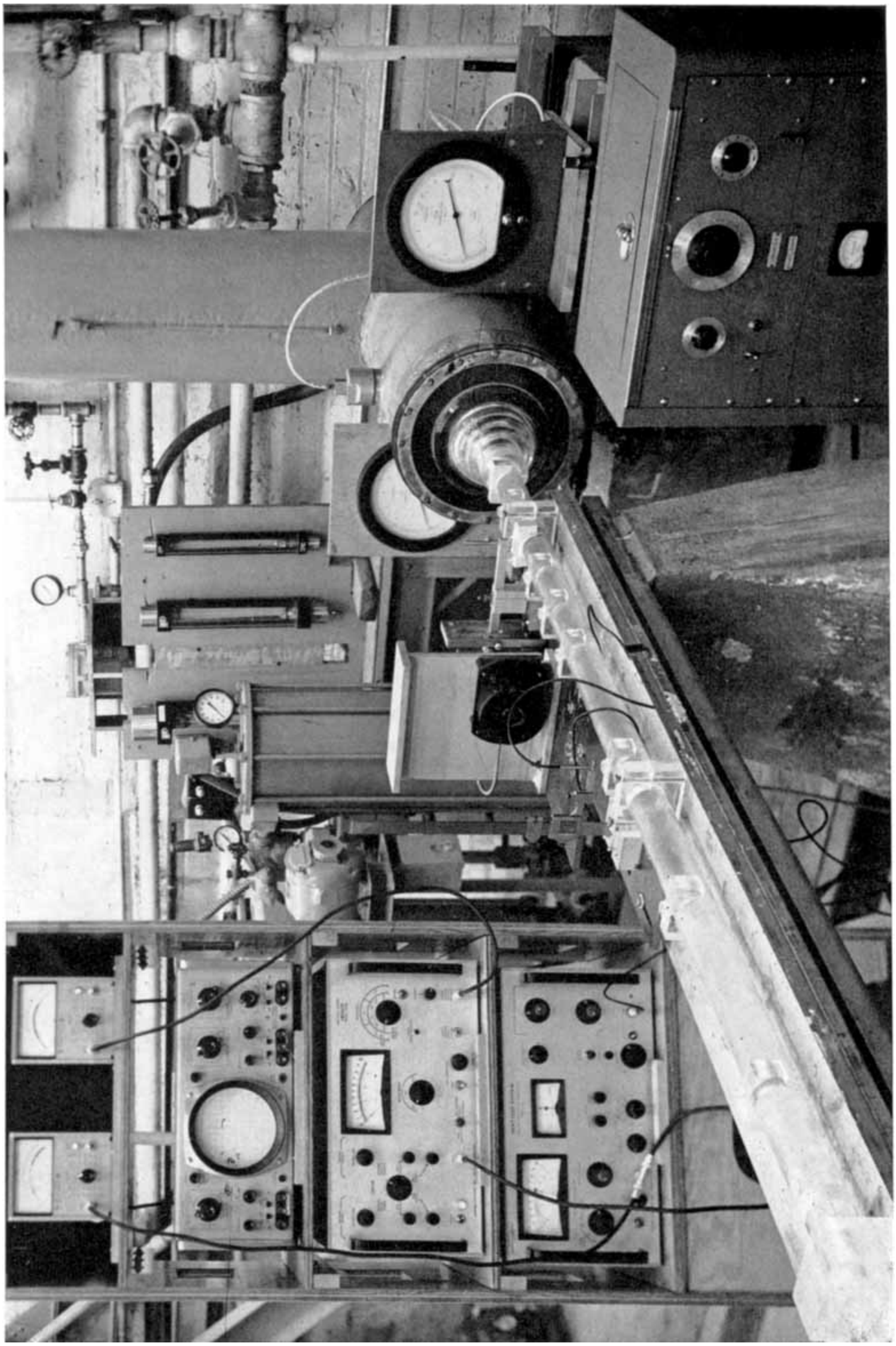

FIGURE 1. Upstream view of the test pipe and the associated equipment.